Period 3, Jan 6, 2024

6. Solve these derivatives/limits

a) $\lim_{x\to -1} \frac{x^2-1}{x+1}$

b) Find $\frac{dy}{dx}$ if $y^3 - 3xy^3 = 2$

c) $y(x) = \sec(e^{3x})$ $\Rightarrow y = \sec e^{u} \Rightarrow y = \sec L$

$u = 3x$ $\quad L = e^{u}$ $\quad \frac{dy}{dL} = \sec L \tan L$

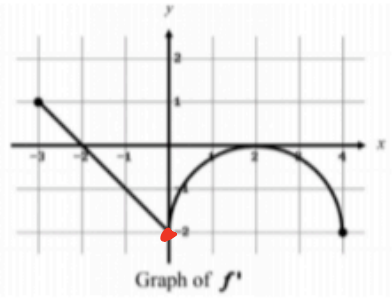
$\frac{du}{dx} = 3$ $\quad \frac{dL}{du} = e^{u}$

$\frac{du}{dx} \cdot \frac{dL}{du} \cdot \frac{dy}{dL}$

$3 \cdot e^{u} \cdot \sec L \tan L = 3 \cdot e^{3x} \sec e^{u} \tan e^{u} = 3 \cdot e^{3x} \sec e^{3x} \tan e^{3x}$

---

3. Let f be a function defined on the closed interval $[-3, 4]$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Point $(0,3)$

Slope at $x=0$ of $F(x) = F'(0) = -2$

$y = mx + b$

$y = -2x + b$

$3 = -2(0) + b$

$b = 3$

$y = -2x + 3$

a) When is $f(x)$ increasing? Justify your responses.

b) When is $f(x)$ decreasing? Justify your responses.

c) Find an equation for the line tangent to the graph of f at the point (0, 3).

Slope and Point

5. If $f(x) = \sqrt{x+2}$, show that the function f satisfies the hypothesis of the Mean Value Theorem on the interval [2, 7]. If it does, find each value of c in (a,b) guaranteed by the theorem.

Mean Value Theorem

Diff / continuous on Interval

not on Interval

$F(x) = \sqrt{x+2} = (x+2)^{\frac{1}{2}}$

$F'(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}} \cdot 1 = \frac{1}{2\sqrt{x+2}}$ $\quad x \neq -2$

MVT

$F(2) = \sqrt{2+2} = \sqrt{4} = 2$

$F(7) = \sqrt{7+2} = \sqrt{9} = 3$

(2,2) (7,3) } Find the slope $\quad \frac{3-2}{7-2} = \frac{1}{5}$

$F'(x) = \frac{1}{2\sqrt{x+2}} = \frac{1}{5}$

$\frac{5}{2} = \frac{2\sqrt{x+2}}{2}$

$\left(\frac{5}{2}\right)^2 = \left(\sqrt{x+2}\right)^2$

$\frac{25}{4} = x+2$ $\quad -2 \quad -2$

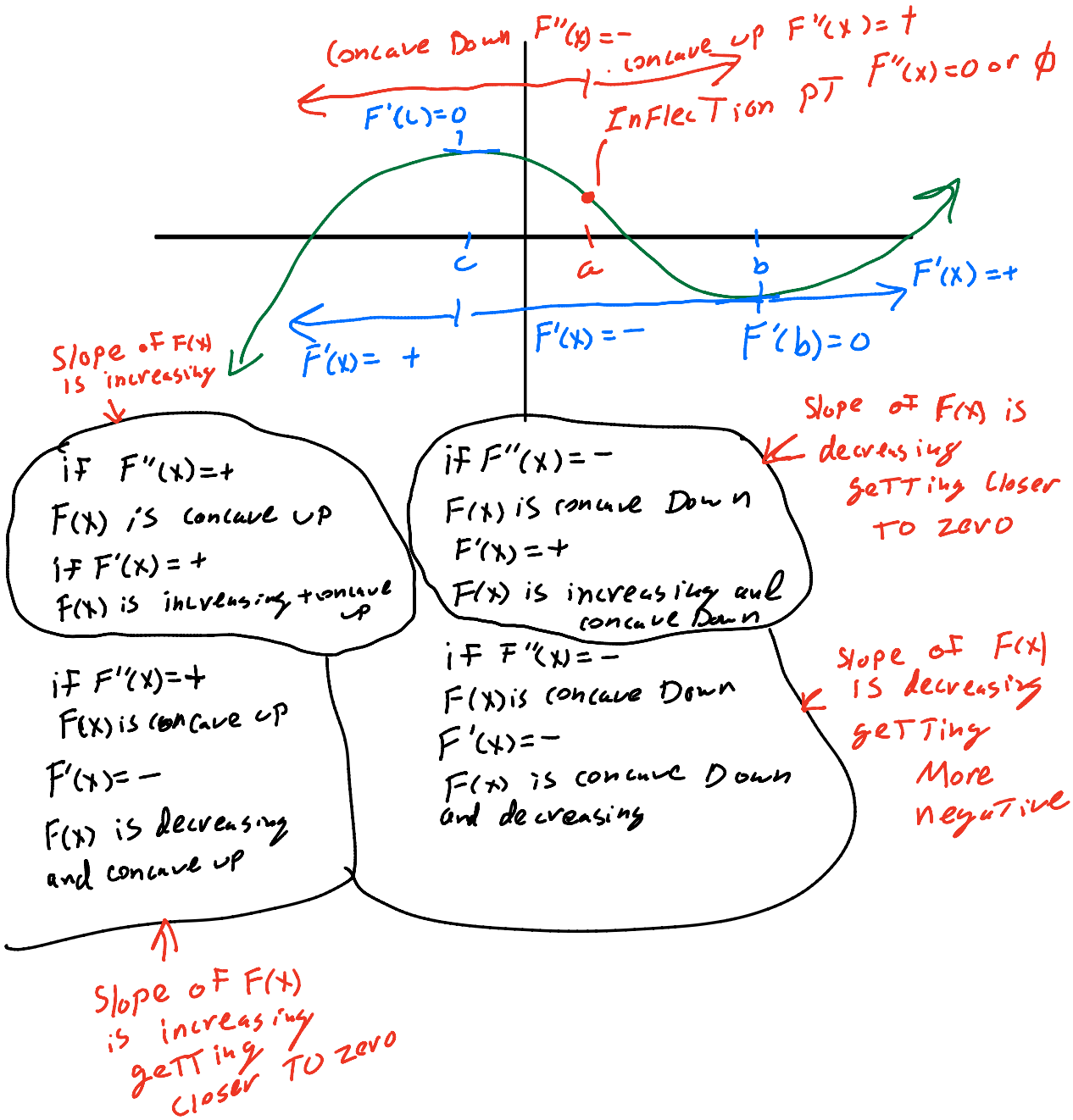
$\frac{25}{4} - \frac{8}{4} = x$

$\frac{17}{4} = x$

$F'\left(\frac{17}{4}\right) = \frac{1}{2\sqrt{\frac{17}{4}+\frac{8}{4}}}$

$\frac{1}{2\sqrt{\frac{25}{4}}}$

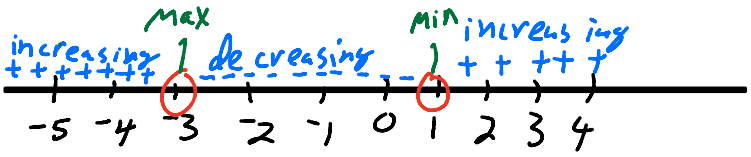
$\frac{1}{2 \cdot \frac{5}{2}} = \frac{1}{5}$


Concave Down
F''(x) = −
Concave up
F''(x) = +
F''(x) = 0 or ϕ
Inflection pt
F'(c) = 0
c
a
b
F'(x) = +
F'(x) = +
F'(x) = −
F'(b) = 0
Slope of F(x) is increasing
if F''(x) = +
F(x) is concave up
if F'(x) = +
F(x) is increasing + concave up
if F''(x) = +
F(x) is concave up
F'(x) = −
F(x) is decreasing and concave up
Slope of F(x) is increasing getting closer to zero
if F''(x) = −
F(x) is concave Down
F'(x) = +
F(x) is increasing and concave Down
Slope of F(x) is decreasing getting closer to zero
if F''(x) = −
F(x) is concave Down
F'(x) = −
F(x) is concave Down and decreasing
Slope of F(x) is decreasing getting More negative

Example 1: Find intervals of increasing and decreasing. Identify the locations of any relative maximum and relative minimum. $y = (x^2 - 3)e^x$

$$\frac{dy}{dx} = 2x \cdot e^x + (x^2-3)\cdot e^x = e^x(2x + x^2 - 3) = e^x(x^2 + 2x - 3)$$

$$\frac{dy}{dx} = e^x(x+3)(x-1)$$



$\frac{dy}{dx} = 0$ when $x = -3, 1$ (critical #'s)

$\frac{dy}{dx} = \emptyset$ never

TEST POINTS INTO $F'(x)$

$F'(-4) = e^{-4}(-4+3)(-4-1) = + \cdot - \cdot - = +$

$F'(0) = e^0(0+3)(0-1) = + \cdot + \cdot - = -$

$F'(2) = e^2(2+3)(2-1) = + \cdot + \cdot +$

increasing $(-\infty, -3) \cup (1, \infty)$

decreasing $(-3, 1)$

To Find Max/mins

Set $F'(x) = 0$ or $\emptyset$

These values of x are your critical #'s

Set up a number line for $F'(x)$

From − to + = Min

From + to − = Max

Student Example 3

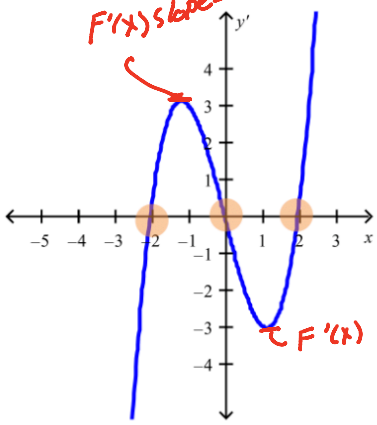
- Identify intervals of increasing/decreasing when you are given f'.
- Identify relative maximum and relative minimum when you are given f'.

Increasing $(-2,0)U(2,\infty)$

Decreasing $(-\infty,-2)U(0,2)$

Rel. Max $at\ x = 0$

Rel. Min $at\ x = -2, 2$



F'(x) slope = 0 = F''(x)

F'(x) slope = 0 = F''(x)

inflection points

F''(x) = 0 or ∅

F''(x) = slope of F'(x)

inflection points are at (−1, +1)

F(x) ⇒ F'(x) is slope of F(x) ⇒ F''(x) concavity of F(x)

F'(x) ⇒ F''(x) is the slope of F'(x) ⇒ F'''(x) concavity of F'(x)

F''(x) ⇒ F'''(x) is the slope of F''(x) ⇒ $F^4(x)$ concavity of F''(x)

Theorem - Points of Inflection

If $(c, f(c))$ is a **point of inflection** of the graph of f, then either $f''(c) = 0$ or f'' does not exist at $x = c$.

True or False:

If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection. Justify why or why not.

FALSE!!

Test $x=-1$ and $x=1$ into $F''(x)=x^2$

$(-1)^2=1=+$

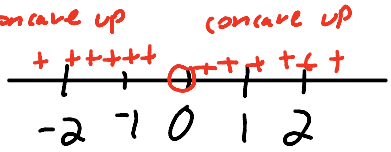
$(1)^2=1=+$

$F(x)=\frac{1}{12}x^4$

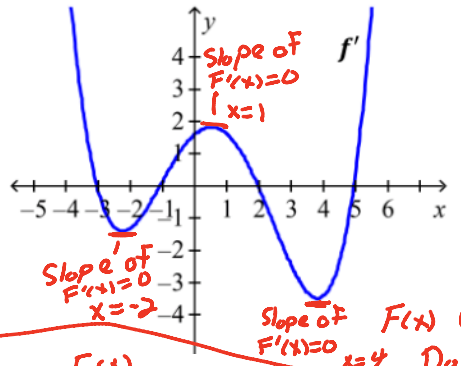
$F'(x)=\frac{1}{3}x^3$

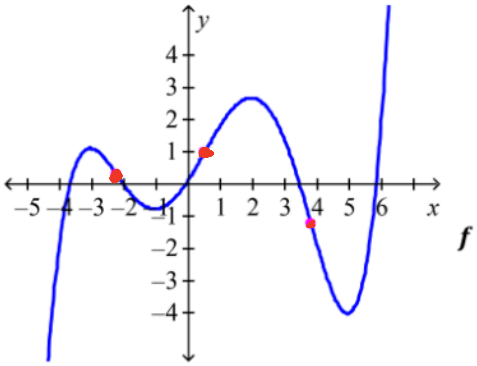
$F''(x)=x^2$

$F''(x)=0=x^2$ when $x=0$



Given the graph of f', (a) determine intervals when f is increasing or decreasing, (b) identify x-values where f has a relative maximum or minimum, and (c) identify intervals where f is concave upward or concave downward (d) Point of Inflection. Justify your answer.





F(x) increasing
$(-\infty,-3) \cup (-1,2) \cup (5,\infty)$

F(x) decreasing
$(-3,-1) \cup (2,5)$

F(x) concave Down
F''(x) = -= Slope of F(x)
$(-\infty,-2) \cup (1,4)$

F(x) concave up
$(-2,1)(4,\infty)$

inflection pts
$x=(-2,1,4)$

AP type question- (d) Identify intervals where f is increasing **and** concave up. Justify $(-1, \frac{1}{2}) \cup (5, \infty)$

$(-1,0.5)U(5,\infty)$ b/c